

Callus Stiffness Evolution during the Healing Process - A mechanical approach

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Abstract— Intramedullary nailing has been used to support long fractured bones, as femur, during the healing process. The load shared between bone/callus and intramedullary nailing is a function of the callus stiffness, that in turn varies during the healing process. Thus, the available mechanical load at callus region during the healing process can define the healing effectiveness. Therefore, the shared load between intramedullary nailing and bone/callus must be accessed. To reach this objective, an analytical model, based in mechanics of solids, is presented to describe the evolution of the load share between bone/callus and intramedullary nailing, at callus region, during the healing process.

Keywords— callus, stiffness, analytical model.

I. INTRODUCTION

Intramedullary nailing, for now on called nailing, has been used to stabilize fractured bones and to shorten the patient recovery time, through enabling the load share between bone and nailing. Intramedullary nails have different types of geometry and applications [1]. The mechanical load must be enough to activate the healing process (callus formation) and therefore get an acceptable bone consolidation. Thus, assessing the level of mechanical load that is disponible to callus region during the healing process, as a result of load sharing with the nailing, is a matter of fundamental importance.

The experimental measuring of biomechanical performance of skeletal systems has been studied by several researches. Bergmann et al. [2] studied how gait patterns can affect the load distribution in human femurs, through the utilization a femoral implant component. Such information was required to test and improve wear, strength and fixation stability of hip implants. Cristofolini et al. [3], focused their work in vitro simulation of the loads occurring in the femur during gait, for hip prostheses stress shielding test purposes. They experimentally measured the bone superficial strains at proximal human femur, generated by the load simulation of each thigh muscle. Experimental methods were used by Edwards et al. [4] to study the internal femoral forces and moments that build up during running, recognizing that the external muscular loading plays an important role in the stress and strain distributions at the external surface of the cortical

bone. Also, the bone surface strains were measured by Simões et al.[5], who studied the influence of muscle action and a horizontally constrained femoral head on the strain distribution, for three loading configurations.

Numerical methods were utilized by researchers as Duda et al. [6] that used F.E. model to show the influence of muscle forces in human femur strain distribution. They demonstrate the importance of an ensemble of muscle forces to reproduce a physiological strain distribution in the femur. Gdoutos et al. [7] besides using numerical methods, as Finite Elements, to review of stress analysis of human femurs, included analytical methods, with the use of the beam theory and experimental methods that incorporate for instance, strain gauges.

Analytical approaches were proposed by Raftopoulos and Qassem et al. [8] which investigate the stresses transmitted to the human femur. Two cases were considered: the femur modeled as three-dimensional isotropic curved beam, and the femur modeled as a three-dimensional anisotropic composite curved beam. Toridis [9] proposed an analytical model for the estimation of the mechanical stresses in a femur under various loading conditions. The femur was treated as a three-dimensional elastic member subjected to axial and shearing forces, bending and twisting moments. In Kenedi and Riagusoff [10] an analytical model was developed to estimate the stress distribution at the external surface of human femur medial cross sections, as an alternative to a more expensive finite element approach. It recognizes cortical and trabecular bone tissues and proposes circular and elliptical geometries to represent real bone cross sections. In Kenedi and Vignoli [11] a straight osteosynthesis plate is analyzed in an analytical bi-material model, to estimate the mechanical stress distributions at plate medial cross sections.

To estimate the load share between bone/callus and nailing during the healing process, several stiffnesses need to be estimated (in function of load, transversal sections and materials) much alike the various approaches showed in previous works [8-11]. With the load shared to bone/callus region results, it is possible to check if it is disponible enough mechanical load to promote a successful bone consolidation. The proposed analytical model is presented in next section.

II. ANALYTICAL MODEL

The analytical model was proposed, based in mechanics of solids, to estimate the load share between bone/callus and nailing during the healing process. The utilization of callus variable stiffness, increasing the callus mechanical properties up to healed bone, differentiates this model from, for instance, the previous ones [8-11].

Fig.1.a shows the model of the femur, with the four forces at proximal region (Taylor's fourth load case of human left femur's head) [12]; and a fixed distal region. Note that although the coordinate systems of Taylor's article and the coordinate system of this work are not the same, the appropriate adaptations were implemented to adequate them. The external forces are: (A) Joint Reaction - P1, (B) Abductors - P2, (C) Iliopsoas - P3 and (D) Ilio-Tibial Tract - P4. See Bitsakos et al. [13] for a more complete muscle loading case description.

These forces localized at proximal femur are recalculated to generate equivalent force and moment equations, at cross section 1 centroid, as shown in detail in Kenedi et al. [14].

$$P_{eq_1} = V_{x_1} \vec{i} + V_{y_1} \vec{j} + V_{z_1} \vec{k} \quad (1)$$

$$M_{eq_1} = M_{x_1} \vec{i} + M_{y_1} \vec{j} + M_{z_1} \vec{k} \quad (2)$$

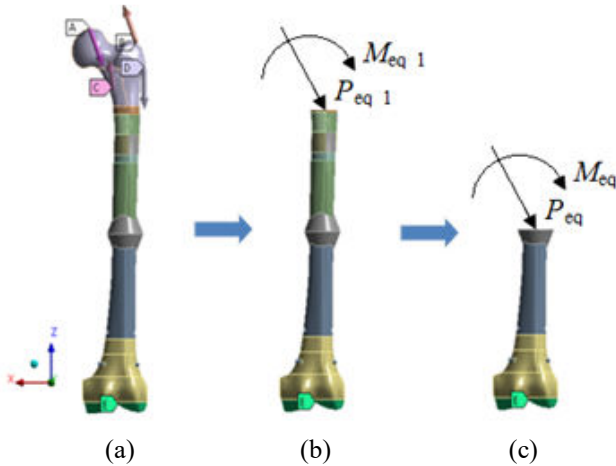


Figure1. Bone/callus and nailing set: (a) with applied forces at proximal region, (b) with the equivalent force and moment at section 1, and (c) with the equivalent force and moment at diaphyseal (callus) section.

Where V_{x_1} , V_{y_1} and V_{z_1} are, respectively, the sum of each force component, of the four external forces, in each direction and M_{x_1} , M_{y_1} and M_{z_1} are, respectively, the sum of each moment component, caused by the four external forces, around each direction.

A. Stiffnesses calculations

With the use of the expressions of Table 1 and the Appendix tables, it is possible to estimate the load share between components of equations (1) and (2), Crandall [15]:

	bone/callus	nailing
Shear force x axis	$V_{x_1}^{bc} = \frac{k_{vx}^{bc}}{k_{vx_eq}} V_{x_1}$	$V_{x_1}^{in} = \frac{k_{vx}^{in}}{k_{vx_eq}} V_{x_1}$
Shear force y axis	$V_{y_1}^{bc} = \frac{k_{vy}^{bc}}{k_{vy_eq}} V_{y_1}$	$V_{y_1}^{in} = \frac{k_{vy}^{in}}{k_{vy_eq}} V_{y_1}$
Axial	$V_{z_1}^{bc} = \frac{k_{vz}^{bc}}{k_{vz_eq}} V_{z_1}$	$V_{z_1}^{in} = \frac{k_{vz}^{in}}{k_{vz_eq}} V_{z_1}$
Bend x axis	$M_{x_1}^{bc} = \frac{k_{bx}^{bc}}{k_{bx_eq}} M_{x_1}$	$M_{x_1}^{in} = \frac{k_{bx}^{in}}{k_{bx_eq}} M_{x_1}$
Bend y axis	$M_{y_1}^{bc} = \frac{k_{by}^{bc}}{k_{by_eq}} M_{y_1}$	$M_{y_1}^{in} = \frac{k_{by}^{in}}{k_{by_eq}} M_{y_1}$
Torsion	$M_{z_1}^{bc} = \frac{k_t^{bc}}{k_{t_eq}} M_{z_1}$	$M_{z_1}^{in} = \frac{k_t^{in}}{k_{t_eq}} M_{z_1}$

Note: See APPENDIX to access the input variables.

Where each equivalent stiffness is calculated at Appendix. The superscript ^{bc} and ⁱⁿ are related, respectively to bone/callus set and nailing. Fig.2 shows the hollow cylinder-based approximation used in the analytical model.

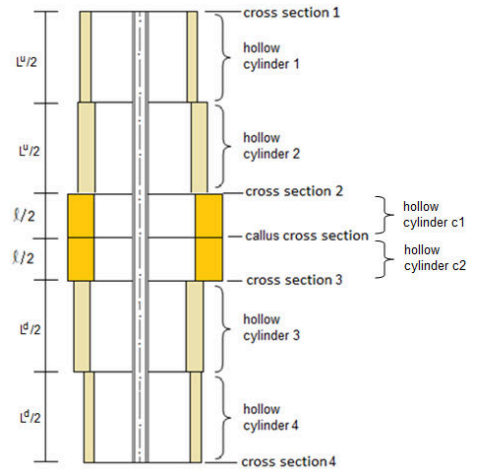


Figure 2. Analytical model, hollow cylinder approximation.

Note that the analytical model, to generate hollow cylinder approximation, supposes that the bone/callus cross sections are formed by hollow circles.

Fig. 3 shows the analytical stiffness model.

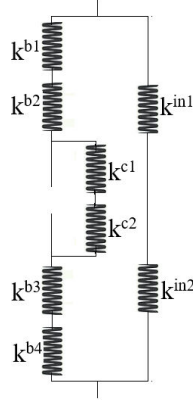


Figure 3. Analytical stiffness model, with superscripts: ^b, for bone, ^c for callus and ⁱⁿ for nailing.

B. Sharing load calculations

It is possible to calculate an equivalent force-couple system from cross section 1 to diaphyseal cross section, already divided in callus and nailing forces and moments:

$$P_{eq}^{\alpha} = V_x^{\alpha} \vec{i} + V_y^{\alpha} \vec{j} + V_z^{\alpha} \vec{k} \quad (3)$$

$$M_{eq}^{\alpha} = M_x^{\alpha} \vec{i} + M_y^{\alpha} \vec{j} + M_z^{\alpha} \vec{k} \quad (4)$$

Where the superscript ^α can be ^{bc} or ⁱⁿ, respectively, bone/callus set and nailing. Table 2 shows the components of equations (3) - (4).

Table 2. Equations (3) and (4) components	
Shear force x axis	$V_x^{\alpha} = V_{x,1}^{\alpha}$
Shear force y axis	$V_y^{\alpha} = V_{y,1}^{\alpha}$
Axial	$V_z^{\alpha} = V_{z,1}^{\alpha}$
Bend x axis	$M_x^{\alpha} = s_y^{\alpha} \cdot V_{z,1}^{\alpha} - s_z^{\alpha} \cdot V_{y,1}^{\alpha} + M_{x,1}^{\alpha}$
Bend y axis	$M_y^{\alpha} = s_z^{\alpha} \cdot V_{x,1}^{\alpha} - s_x^{\alpha} \cdot V_{z,1}^{\alpha} + M_{y,1}^{\alpha}$
Torsion	$M_z^{\alpha} = s_x^{\alpha} \cdot V_{y,1}^{\alpha} - s_y^{\alpha} \cdot V_{x,1}^{\alpha} + M_{z,1}^{\alpha}$

Note: See APPENDIX to access the input variables.

$s^{\alpha} = s_x^{\alpha} \vec{i} + s_y^{\alpha} \vec{j} + s_z^{\alpha} \vec{k}$ are position vectors between the centroids of 1 and diaphyseal cross sections (callus and nailing cross sections centroids).

C. Analytical Model Data

The four external forces components and the distances between forces point of application and the cross section set 1 centroid are shown in Table 3.

Table 3 - External forces and respective distances to centroid 1 [16]

	forces P_n (N)	distance d_n (mm)
1	(-1,062; -130; -2,800)	(50.6; 0.7; 98)
2	(430; 0; 1,160)	(-13.6; -3.1; 80)
3	(78; 560; 525)	(18.7; -25.9; 23.7)
4	(0; 0; -1,200)	(-24.7; -0.8; 48)

The vertical distance z between cross section 1 and diaphyseal (callus cross section of Fig. 2) is 140 mm. The nailing cross-sections geometry is defined: $r_{i, in} = 2.5$ mm and $r_{o, in} = 5$ mm. $A_{in} = 5.89 \cdot 10^{-5} \text{ m}^2$ and $I_{in} = 4.6 \cdot 10^{-10} \text{ m}^4$ [16]. Table 4 shows the geometrical properties of bone/callus and Table 5 shows the mechanical properties of bone and nailing.

Table 4. Geometrical properties of bone/callus [16]

hollow cylinders	$r_{i, n}$ (mm)	$r_{o, n}$ (mm)	L_n (mm)	A_n (10^{-4} m^2)	I_n (10^{-8} m^4)
1	9.5	19	62.5	8.51	9.6
2	7.9	19	62.5	9.38	9.9
c1	9.5	21.5	15	11.7	16.1
c2	9.5	21.5	15	11.7	16.1
3	11	17.4	62.5	5.71	6.0
4	13.4	30	62.5	22.6	61.1

See Fig. 2 to check the hollow cylinders denomination.

Note that callus external radius $r_{o, c1}$ and $r_{o, c2}$ where obtained increasing the diaphyseal cortical bone external radius 18.2 mm by 18%. Also, by an analytical model limitation, $I_{n, x} = I_{n, y} = I_n$ (because the bone/callus cross sections are approximate by hollow circles) and the polar moment of inertia of area was calculated by the following approximation: $J_n = I_{n, x} + I_{n, y} = 2 \cdot I_n$.

Table 5. Mechanical properties of bone and nailing [16].

mechanical property	Cortical bone	Titanium
E (GPa)	15.7	105
ν	0.325	0.34

Where E is the Young Modulus and ν is the Poisson ratio. The shear modulus was estimated using the following approximation $G = 1/(2 \cdot (1 + \nu))$. The callus mechanical properties were varied from a very soft material up to cortical bone mechanical properties, in GPa, ($0.1 \leq E_c \leq E_b$ and $0.1 \leq G_c \leq G_b$).

In the next section the results of the analytical model are presented.

III. RESULTS

With the use of mathematical software, as Mathcad, it is possible to implement the analytical model equations varying the callus mechanical properties to simulate the stiffness increase during the healing process. The bone was modeled as linear, elastic and isotropic.

Figures 4-6 show the forces and moments sharing results through the simulation of the healing process, obtained implementing the analytical model equations in the Mathcad software.

Fig.4 show the sharing load results of transversal forces V_x and V_y .

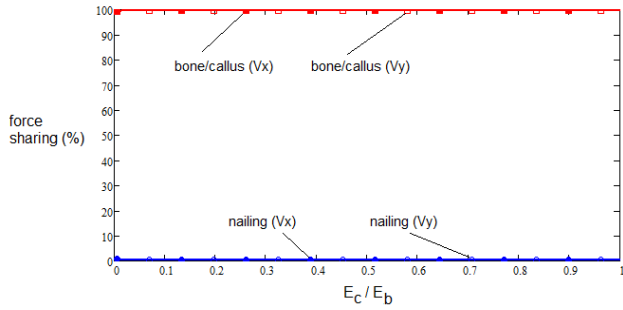


Figure 4. Evolution of load share through healing process for transversal forces V_x and V_y .

Fig.5 show the sharing load results of longitudinal forces V_z .

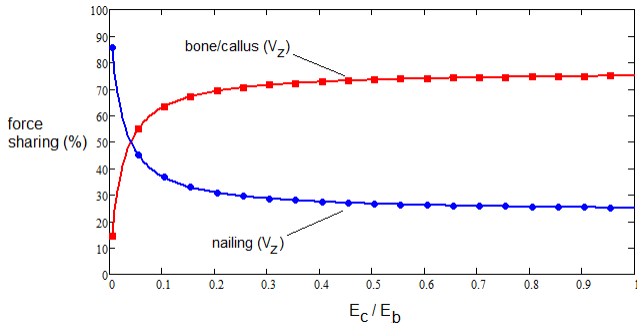


Figure 5. Evolution of load share through healing process for longitudinal force V_z .

The graphic results of Figures 4 and 5 show the forces load share between bone/callus and nailing. As the healing process progresses the callus became stiffer, by the increase of its mechanical properties up to healed bone ones. Note that the load share of transversal forces V_x and V_y of Fig.4 have very distinct performance, where the bone/callus is responsible to resist almost all the transversal loads.

On the contrary, Fig. 5 shows the load share of longitudinal force V_z . At the beginning of the healing process, when callus mechanical properties are quite low, the nailing resist an order of magnitude more than the bone/callus set. With the continuation of the healing process the mechanical properties of callus region improves continuously, until they are sufficient to turn bone/callus stiffer than nailing, as show in early stages of Fig. 5. For this example, at the end of healing process, the healed bone resists around three quarters of the axial load whereas the nailing resists the remaining quarter.

Fig.6 show the sharing load results of moments M_x and M_y in Fig. 6.a and M_z in Fig. 6.b.

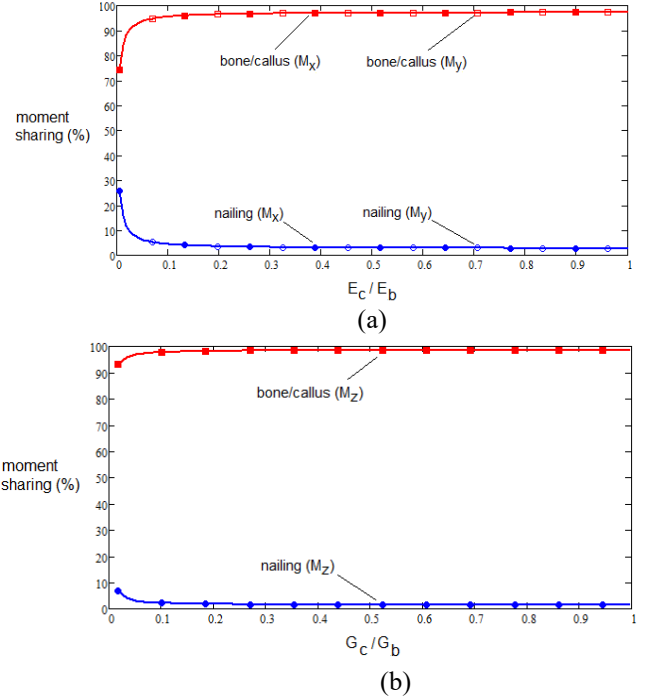


Figure 6. Evolution of load share through healing process for: (a) bending moments M_x and M_y (b) torcional moment M_z .

The graphic results of Fig. 6 show the moments load share between bone/callus and nailing, as the healing process progresses, by stiffening the callus mechanical properties up to healed bone ones. Note that the load share for both, bending moments M_x and M_y of Fig. 6.a and torcional moment M_z of Fig. 6.b; the bone/callus are responsible to resist almost all the moments. Note that this behavior is reinforced as the healing process goes on.

Note that in Figures 4-6 the time variable is not accessed. It is considered a limitation of this proposed model.

IV. CONCLUSIONS

An analytical model was presented, based in mechanics of solids, to estimate the influence of the stiffness variation of callus during the healing process, in the sharing load between bone/callus and nailing. There were estimate twelve stiffnesses, one for each loading (V_x , V_y , V_z , M_x , M_y and M_z) referent to each material (bone/callus and nailing). These stiffnesses were used to estimate the load sharing between bone/callus and nailing during the healing process.

The results show that, for this example, that the bone/callus is stiffer that the nailing for almost all loading, except for longitudinal loading V_z . In this case the nailing at the beginning is stiffer that bone/callus, but as the callus increases its mechanical properties, through healing process, the bone/callus turns stiffer than the nailing. At the end of the healing process, the healed bone resists around three quarters of the axial load, leaving the nailing to resists the remaining quarter.

Although this analytical model doesn't access the time variable, it brings quantitative information about how the various stiffnesses behave during the healing process. The longitudinal load level that could reach by the healed bone at end of the healing process can be checked with the minimum level of load, or even strains, necessary to reach an effective remodeling phase. This information can aid in the decision of an appropriate nailing selection prior to its implantation.

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CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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APPENDIX

Table A.1. Bone stiffnesses

load	Bone (two cylinders in series)
Shear force x axis*	$k_{vx}^{b,ud} = \frac{24E^b}{(L^{ud})^3} \left(\frac{1}{I_{x,\delta 1}^b} + \frac{7}{I_{x,\delta 2}^b} \right)^{-1}$
Shear force y axis*	$k_{vy}^{b,ud} = \frac{24E^b}{(L^{ud})^3} \left(\frac{1}{I_{y,\delta 1}^b} + \frac{7}{I_{y,\delta 2}^b} \right)^{-1}$
Axial	$k_{vz}^{b,ud} = \frac{2E^b}{L^{ud}} \left(\frac{1}{A_{z,\delta 1}^b} + \frac{1}{A_{z,\delta 2}^b} \right)^{-1}$
Bend x axis	$k_{bx}^{b,ud} = \frac{2E^b}{L^{ud}} \left(\frac{1}{I_{x,\delta 1}^b} + \frac{1}{I_{x,\delta 2}^b} \right)^{-1}$
Bend y axis	$k_{by}^{b,ud} = \frac{2E^b}{L^{ud}} \left(\frac{1}{I_{y,\delta 1}^b} + \frac{1}{I_{y,\delta 2}^b} \right)^{-1}$
Torsion	$k_t^{b,ud} = \frac{2G^b}{L^{ud}} \left(\frac{1}{J_{\delta 1}^b} + \frac{1}{J_{\delta 2}^b} \right)^{-1}$

(*) It is supposed that the shear forces V_x and V_y were only resisted by bending moment stiffnesses.

Table A.2. Callus and nailing stiffnesses

load	Callus (two cylinders in series)	Nailing
Shear force x axis*	$k_{vx}^c = \frac{24E^c}{(l)^3} \left(\frac{1}{I_{x,1}^c} + \frac{7}{I_{x,2}^c} \right)^{-1}$	$k_{vx}^{in} = \frac{3E^{in}I_x^{in}}{(L)^3}$
Shear force y axis*	$k_{vy}^c = \frac{24E^c}{(l)^3} \left(\frac{1}{I_{y,1}^c} + \frac{7}{I_{y,2}^c} \right)^{-1}$	$k_{vy}^{in} = \frac{3E^{in}I_y^{in}}{(L)^3}$
Axial	$k_{vz}^c = \frac{2E^c}{l} \left(\frac{1}{A_{x,1}^c} + \frac{1}{A_{x,2}^c} \right)^{-1}$	$k_{vz}^{in} = \frac{A^{in}E^{in}}{L}$
Bend x axis	$k_{bx}^c = \frac{2E^c}{l} \left(\frac{1}{I_{x,1}^c} + \frac{1}{I_{x,2}^c} \right)^{-1}$	$k_{bx}^{in} = \frac{E^{in}I_x^{in}}{L}$
Bend y axis	$k_{by}^c = \frac{2E^c}{l} \left(\frac{1}{I_{y,1}^c} + \frac{1}{I_{y,2}^c} \right)^{-1}$	$k_{by}^{in} = \frac{E^{in}I_y^{in}}{L}$
Torsion	$k_t^c = \frac{2G^c}{l} \left(\frac{1}{J_1^c} + \frac{1}{J_2^c} \right)^{-1}$	$k_t^{in} = \frac{J^{in}G^{in}}{L}$

(*) It is supposed that the shear forces V_x and V_y were only resisted by bending moment stiffnesses.

Where, the superscript ^b and ^c are related, respectively, with bone and callus, the superscript ^{ud} is substituted, respectively by ^u for the up cylinders and by ^d for the down cylinders. The subscripts _{δ1} and _{δ2} are substituted, respectively by ₁ and ₂ for the up cylinders and by ₃ and ₄ for the down cylinders. L (where, $L = L^u + l + L^d$) is the length of bone/callus set, that is the same as the length of the nailing, l is the length of callus. Note that $L^u = L_1 + L_2$, $l = L_{c1} + L_{c2}$ and $L^d = L_3 + L_4$. A is the transversal area, I is the moment of inertia of area, J is the polar moment of inertia of area, E is the elastic modulus, G is the shear modulus and k is the stiffness.

Table A.4. Equivalent stiffnesses

load	Equivalent stiffnesses
Shear force x axis*	$k_{vx_eq} = k_{vx}^{bc} + k_{vx}^{in}$
Shear force y axis*	$k_{vy_eq} = k_{vy}^{bc} + k_{vy}^{in}$
Axial	$k_{vz_eq} = k_{vz}^{bc} + k_{vz}^{in}$
Bend x axis	$k_{bx_eq} = k_{bx}^{bc} + k_{bx}^{in}$
Bend y axis	$k_{by_eq} = k_{by}^{bc} + k_{by}^{in}$
Torsion	$k_{t_eq} = k_t^{bc} + k_t^{in}$

Table A.3. Partial equivalent stiffnesses

load	Bone/Callus (in series)	Nailing
Shear force x axis*	$k_{vx}^{bc} = \left(\frac{1}{k_{vx}^{b,u}} + \frac{1}{k_{vx}^c} + \frac{1}{k_{vx}^{b,d}} \right)^{-1}$	k_{vx}^{in}
Shear force y axis*	$k_{vy}^{bc} = \left(\frac{1}{k_{vy}^{b,u}} + \frac{1}{k_{vy}^c} + \frac{1}{k_{vy}^{b,d}} \right)^{-1}$	k_{vy}^{in}
Axial	$k_{vz}^{bc} = \left(\frac{1}{k_{vz}^{b,u}} + \frac{1}{k_{vz}^c} + \frac{1}{k_{vz}^{b,d}} \right)^{-1}$	k_{vz}^{in}
Bend x axis	$k_{bx}^{bc} = \left(\frac{1}{k_{bx}^{b,u}} + \frac{1}{k_{bx}^c} + \frac{1}{k_{bx}^{b,d}} \right)^{-1}$	k_{bx}^{in}
Bend y axis	$k_{by}^{bc} = \left(\frac{1}{k_{by}^{b,u}} + \frac{1}{k_{by}^c} + \frac{1}{k_{by}^{b,d}} \right)^{-1}$	k_{by}^{in}
Torsion	$k_t^{bc} = \left(\frac{1}{k_t^{b,u}} + \frac{1}{k_t^c} + \frac{1}{k_t^{b,d}} \right)^{-1}$	k_t^{in}